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Also, as regards the inverse problem, what do we know about the factors of $N \equiv b\cdot 2^b + 1$, when b is a known positive integer, e. g., $b = 141$.

2728. Proposed by NORMAN ANNING, Somewhere in France.

A material triangle of uniform density and thickness is of such a shape that when suspended from the vertices in succession, the lower sides have slopes of $1:1$, $1\frac{1}{2}:1$, and $3:1$. Construct the triangle given that the shortest side is 10 inches.

By definition, an $a:1$ slope makes an angle with the vertical whose tangent is a.

2729. Proposed by N. P. PANDYA, Sojitra, India.

Solve in integers $x^3 + 3y^4 = z^2$.

2730. Proposed by W. E. HEAL, Washington, D. C.

If $z^n = x^n + y^n$, where x, y, z, n are integers, and n prime, prove that

$$\begin{aligned}\{z^{(n-1)/2}[(z - x)x^{(n-1)/2} + (z + y)y^{(n-1)/2}]\}^2 &+ \{x^{(n-1)/2}[2x^{(n+1)/2} + y^{(n-1)/2}(x - y)]\}^2 \\ &= \{z^{(n-1)/2}[(z + x)x^{(n-1)/2} + (z - y)y^{(n-1)/2}]\}^2 + \{y^{(n-1)/2}[2y^{(n+1)/2} - x^{(n-1)/2}(x - y)]\}^2 \\ &= [z^{n+1} + (x^{(n+1)/2} - y^{(n+1)/2})^2] \times [z^{n-1} + (x^{(n-1)/2} + y^{(n-1)/2})^2].\end{aligned}$$

Also prove that $z^{n+1} + (x^{(n+1)/2} - y^{(n+1)/2})^2$ and $z^{n-1} + (x^{(n-1)/2} + y^{(n-1)/2})^2$ have no common factors except the divisors of $z^{n-1}(z + x)(z + y)$.

2731. Proposed by JAMES K. WHITTEMORE, Yale University.

A bowl is in the form of a paraboloid of revolution. If for a given volume the surface is a minimum, prove that the ratio of the diameter of the top to the depth is approximately 1.86.

SOLUTIONS OF PROBLEMS.

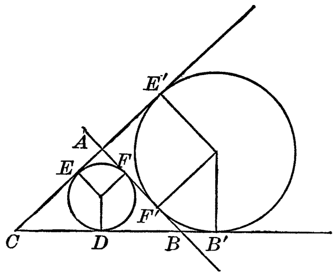
2660. Proposed by JOSEPH E. ROWE, State College, Pa.

Prove that the distance measured along the side of a triangle, from the point of contact with the inscribed and escribed circle, is equal to the side of the triangle between the two circles.

Solution by George F. Wilder, Brooklyn, N. Y.

Let ABC be the triangle and s its semi-perimeter. From the equalities $AE = AF$, $CE = CD$, and $BD = BF$, we have $s = AE + CD + BD$, or $s = AE + a$, since $CD + BD = a$. Hence, $AE = s - a$, (1). Also, since $AE' = AF'$ and $BF' = BD'$, $CE' + CD' = 2s$. Hence, $CD' = s$, since $CE' = CD'$. Hence, $BD' = CD' - CB = s - a$, (2). From (1) and (2) we have $AE = BD'$.

Now $EE' = EA + E'A = BD' + AF'$, since $AE = BD'$ and $AE' = AF'$, and hence $EE' = BF' + AF' = AB$.



Similar and various other solutions were received from the following contributors: J. V. Balch, Horace Olson, Paul Capron, S. W. Reeves, C. P. Sousley, J. F. Connelly, Elijah Swift, and the Proposer.

This problem was proposed in an examination test in Cooper Union, New York, and solutions similar to the one above were received from G. J. Harris, H. F. Mathussen, I. J. Fajans, C. J. Schmitt, H. Gladstone, Morris Devorken, Angelo J. Sajin, David Tennenbaum, Jacob Greismann, I. Millenky, MAX SILVERMAN, G. VANDERBOON, J. LEBEDEFF, P. WEINBERG, MOSE KAHN, MORRIS GROSS, PETER PUMO, HYMAN MILBERG, S. L. RADER, O. C. SIMONSEN, ABRAHAM PLETMAN, FRANK PATLI, M. J. MONTFERRANTE, JOSEPH MENT, MORRIS BUNSIS, SAMUEL COHEN, and CLIFFORD STRAIN.

2668. Proposed by B. F. FINKEL, Drury College.

Show that

$$v = \frac{2}{9}\frac{ga^2\sigma}{\eta},$$

where a is the radius of a droplet, σ its density, η the viscosity of the air and v the velocity under gravity g. *Stokes's law.*

SOLUTION BY ALBERT R. NAUER, St. Louis, Missouri.

Stokes finds analytically a formula determining the resistance of a fluid medium to the movement of a sphere through it,

$$F = \tfrac{4}{3}\pi\rho an\sqrt{-1}[af_1'(a) + 2f_2(a)]e^{\sqrt{-1}nt}, \tag{I}$$

Stokes's *Mathematical and Physical Papers*, Vol. III, p. 33. From a previous development (p. 28) is given

$$f_1''(r) - \frac{2}{r^2}f_1(r) = 0, \qquad \text{and} \qquad f_2''(r) - \frac{2}{r^2}f_2(r) - m^2f_2(r) = 0. \tag{II}$$

Then the equations of condition become on putting $f(r)$ for $f_1(r) + f_2(r)$, and $r = a$,

$$f'(a) = ac, \qquad f(a) = \tfrac{1}{2}a^2c, \tag{1}$$

$$f'(b) = 0, \qquad f(b) = 0. \tag{2}$$

The integration of the differential equations (II) has no difficulties, the first one is of a well-known form and the second one is a Riccati equation solvable in finite terms.

The integrals are

$$f_1(r) = \frac{A}{r} + Br^2 \qquad \text{and} \qquad f_2(r) = Ce^{-mr}\left(1 + \frac{1}{mr}\right) + De^{mr}\left(1 + \frac{1}{mr}\right).$$

Here $D = 0$, since otherwise the velocity would be infinite at an infinite distance; $B = 0$, otherwise the velocity would be finite when $r = \infty$.

From Eq. (1), we get

$$A = \tfrac{1}{2}a^3c - aCe^{-ma}\left(1 + \frac{1}{ma}\right).$$

Putting $b = \infty$, we get from Eq. (2),

$$\frac{d}{da}\{\text{Equation (1)}\} = 0 = \frac{3a^2c}{2} - aC(1 - m - 1)e^{-ma}, \qquad C = \frac{3ac}{2m}e^{ma},$$

and then

$$A = \tfrac{1}{2}a^3c + \frac{3a^2c}{2m}\left(1 + \frac{1}{ma}\right).$$

Substituting in (I),

$$F = -\tfrac{2}{3}\pi a^3cn\sqrt{-1}\left(1 + \frac{9}{ma} + \frac{9}{m^2a^2}\right)e^{\sqrt{-1}nt}. \tag{III}$$

Substituting for m its value, which is the particular square root of $(n\sqrt{-1})/\mu'$ which has its real part positive, and write V for $ce^{\sqrt{-1}nt}$. Then

$$F = -6\pi\mu'\rho aV, \tag{IV}$$

where $\mu' = \mu/\rho$, called by Stokes, the *index of friction* and ρ is the density of the fluid. Whenever the motion is so slow that the part of the resistance which depends on the square of the velocity may be neglected, we have, supposing V to be the terminal velocity;

$$-F = \tfrac{4}{3}\pi ga^3(\sigma - \rho),$$